Signal Processing

Week 2.3

Monday, Nov. 25, 2024

Christian Tiberius

TU Delft

MUDE

Modelling, Uncertainty and Data for Engineers

MUDE - Week 2.3: Signal Processing

- We will add the dimension of **time** to inputs to models, and to observations.
- We will study signals:
  - *A **signal**, as a function of one or more variables, may be defined as an observable change in a quantifiable entity**
- If the independent variable is *time*, *signal* = *time series*
- We cover time series analysis (week 2.4).
- Week 2.3 entails the study of time-varying signals in the **frequency domain**.

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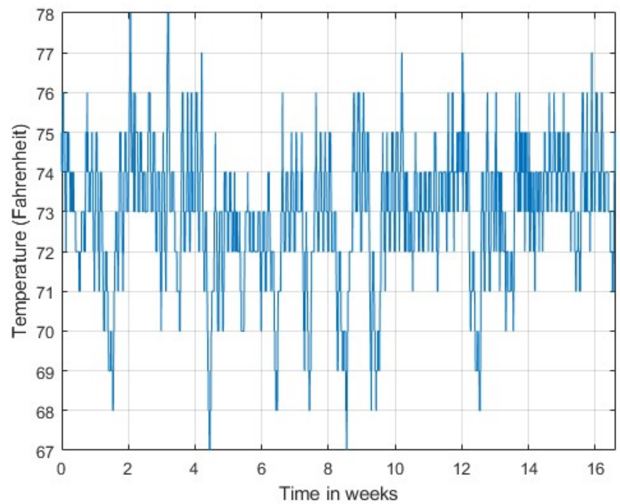
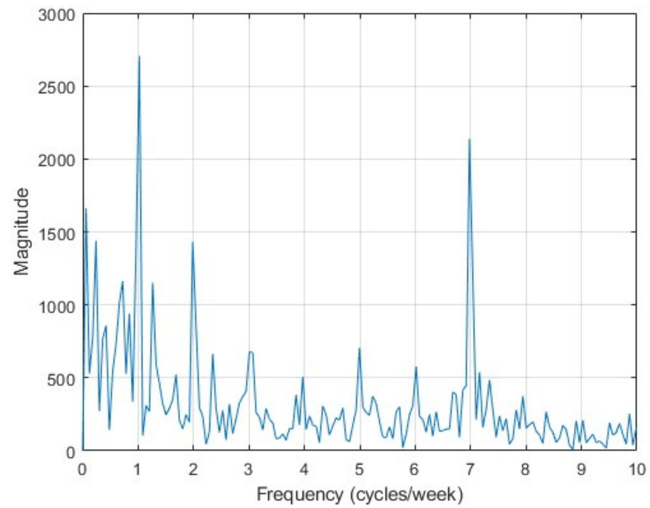
* Pragnan Chakravorty, "What Is a Signal? [Lecture Notes]," IEEE Signal Processing Magazine, vol. 35, no. 5, pp. 175-177, Sept. 2018. doi: 10.1109/MSP.2018.2832195

Why the frequency domain?

- It allows to observe several characteristics of the signal that are either *not easy to see*, or *not visible at all* when you look at the signal in the time domain.

- For instance, frequency-domain analysis becomes useful when you are looking for *cyclic behavior* of signals.

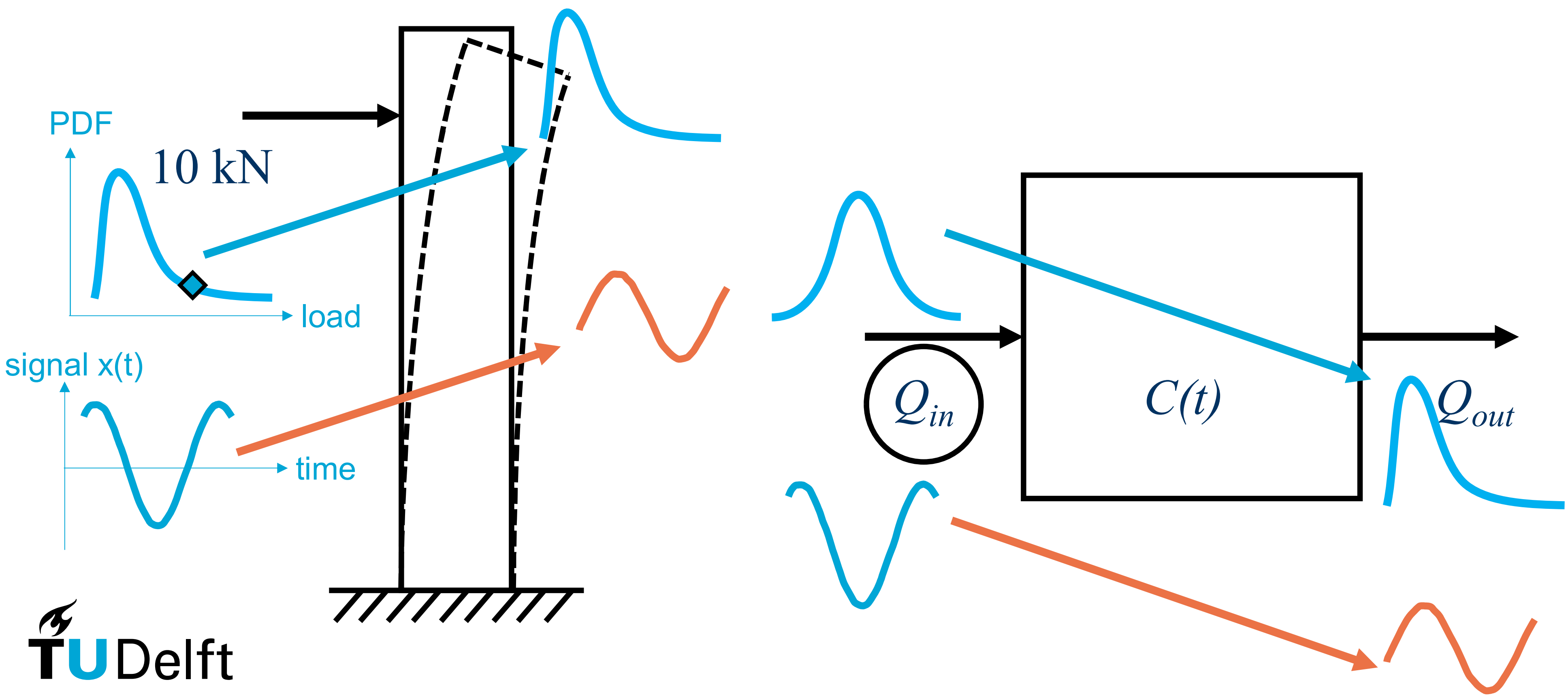




Credits: Mathworks.com

related to risk & reliability in MUDE Q1

deterministic design – load often given in assignment



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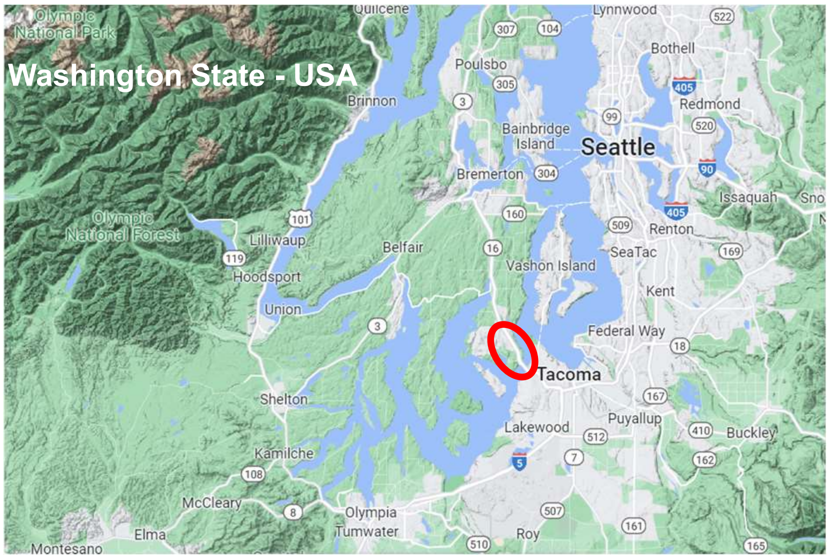
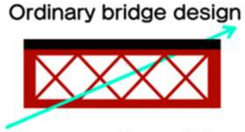
Tacoma Narrows Bridge

also known as 'Galloping Gertie' …

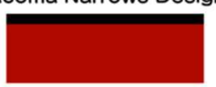


Ordinary bridge design



Wind can pass through trusses

Tacoma Narrows Design



Wind would be forced around trusses

https://www.youtube.com/watch?v=y0xohjV7Avo

video by Smithsonian National Air and Space Museum

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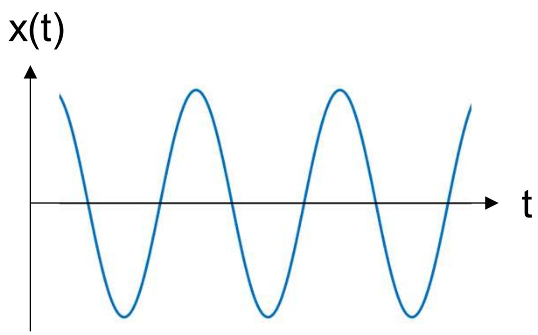
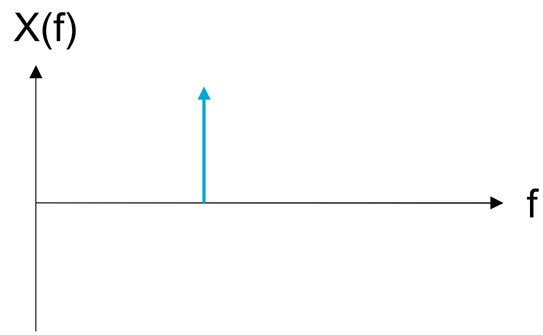
signal: time and frequency domain

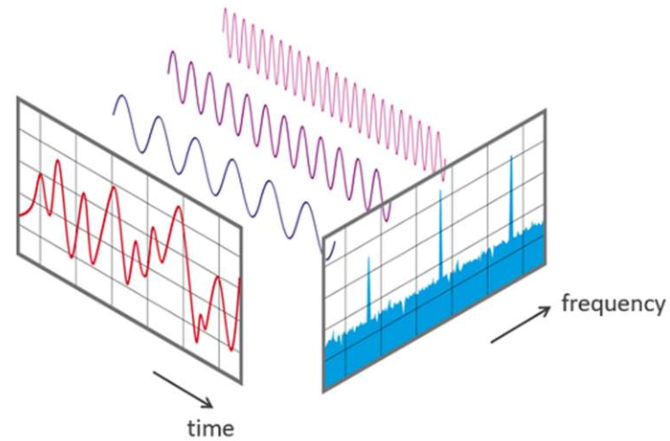
two different view-points on the same phenomenon:

time-domain

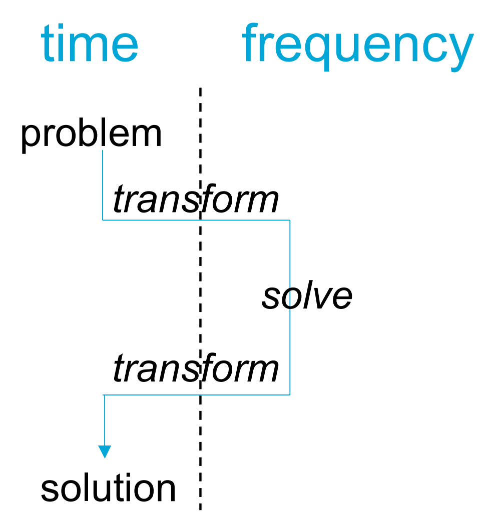


frequency-domain





solution strategy in practice



frequency domain offer extra tools for the engineer

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transforming Differential Equation into frequency domain (optional)

1st order DE: $\frac{1}{k}\frac{dy(t)}{dt} + y(t) = x(t)$

transform from time to frequency domain: $\frac{1}{k}\, j2\pi f\; Y(f) + Y(f) = X(f)$

reworking into: $H(f) = \frac{Y(f)}{X(f)} = \frac{k}{j2\pi f +}$, which is system frequency response

transform back to time domain $h(t) = ke^{-kt}u(t)$, which is system impulse response

$k > 0$, $u(t)$ step response

$u(t) = 1 \text{ for } t \geq 0$

now compute output $y(t)$ given input $x(t)$:

convolution: $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\lambda)ke^{-k(t-\lambda)}u(t-\lambda)d\lambda$

for instance with $x(t) = u(t)$ we find $y(t) = \int_0^t ke^{-k(t-\lambda)}d\lambda = 1 - e^{-k}$ with $t \geq 0$

solving Differential Equation in time domain (optional)

short note on solving 1st-order differential equation - worked example

Christian Tiberius

November 2022

1 Introduction

This short note demonstrates, by means of an example, how to solve, analytically, in the time domain, a basic first order differential equation.

The example, and derivation, is taken from 'Signals and systems - continuous and discrete' by R.E. Ziemer, W.H. Tranter and D.R. Fannin, Prentice Hall, 4th edition, 1998 (Example 2-1).

2 First order differential equation

The differential equation is given as

$$\frac{1}{k}\frac{dy(t)}{dt} + y(t) = x(t) \tag{1}$$

with input $x(t)$ and output $y(t)$.

The goal of this exercise is to express output $y(t)$ (explicitly) in terms of input $x(t)$.

We assume that $x(t)$ is applied at time $t = t_0$ and that $y(t_0) = y_0$.

3 Homogeneous solution

The solution to the homogeneous differential equation

$$\frac{1}{k}\frac{dy(t)}{dt} + y(t) = 0 \tag{2}$$

is found by assuming a solution of the form $y(t) = Ae^{pt}$, and substituting this in the above homogeneous differential equation leads to $p = -k$. Hence, the homogeneous solution reads

$$y(t) = Ae^{-kt} \tag{3}$$

4 Total solution

In order to find the total solution we use the technique of 'variation of parameters', which consists of assuming a solution of the form of the above homogeneous solution, but with undetermined coefficient A replaced by a function of time $A(t)$ which is to be found. Hence, we assume that

$$y(t) = A(t)e^{-kt} \tag{4}$$

Differentiating (and using the chain-rule), leads to

$$\frac{dy(t)}{dt} = \left(\frac{dA(t)}{dt} - kA(t)\right)e^{-kt} \tag{5}$$

Next, substituting the assumed solution (4) and its derivative (5) in the original differential equation (1), we obtain

$$\frac{1}{k}e^{-kt}\frac{dA(t)}{dt} = x(t)$$

or

$$\frac{dA(t)}{dt} = x(t)\,ke^{kt} \tag{6}$$

Solving for $\frac{dA(t)}{dt}$, i.e. integrating the above expression, yields

$$A(t) - A(t_0) = k\int_{t_0}^{t} x(\lambda)e^{k\lambda}d\lambda$$

and using (4) at time t_0: $y(t_0) = y_0 = A(t_0)e^{-kt_0}$, or $A(t_0) = y_0\,e^{kt_0}$, so we find the varying parameter $A(t)$ as

$$A(t) = k\int_{t_0}^{t} x(\lambda)e^{k\lambda}d\lambda + y_0\,e^{kt_0} \tag{7}$$

and this can be substituted in the assumed solution (4) and this yields

$$y(t) = y_0\,e^{-k(t-t_0)} + k\int_{t_0}^{t} x(\lambda)e^{-k(t-\lambda)}d\lambda$$

Assuming that the input $x(t)$ is applied at $t = -\infty$, hence $t_0 = -\infty$, and that $y_0 = y(t_0) = y(t = -\infty) = 0$, we obtain

$$y(t) = \int_{-\infty}^{t} x(\lambda)ke^{-k(t-\lambda)}d\lambda \tag{8}$$

5 Solution

Now the output $y(t)$ to input $x(t)$ can be found through solving the above integral.

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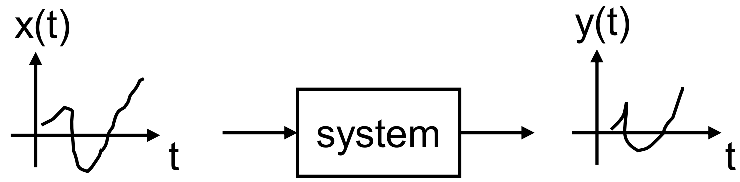
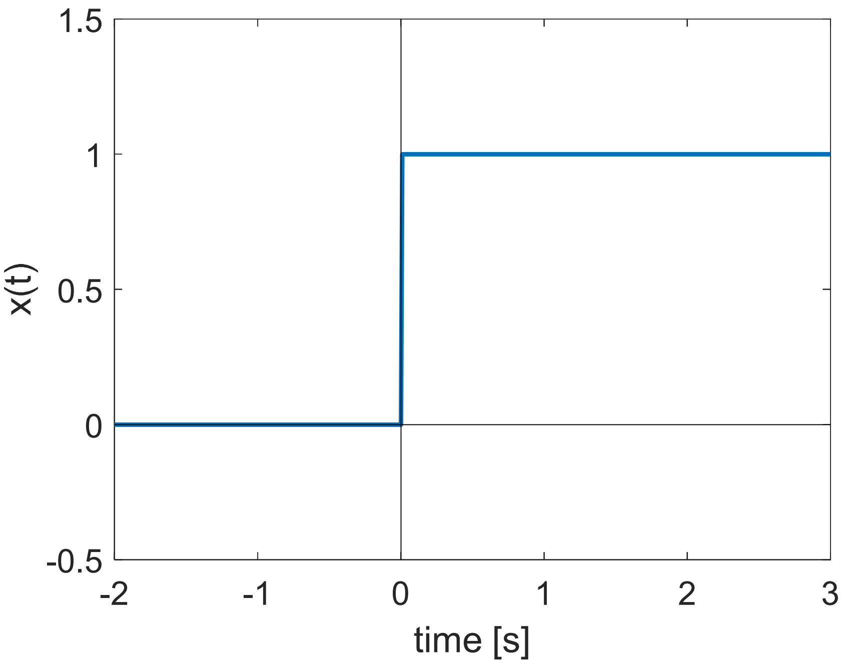
system: input - output





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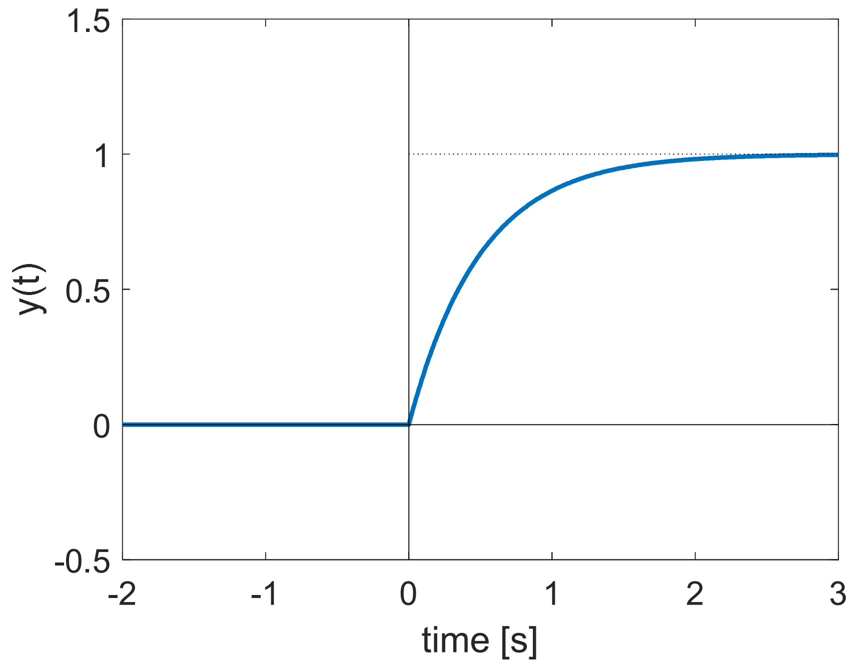

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sound demo

Signal Processing with audio

Author: Steven Lin
Date: 21.10.2022
Reference: Music in Python by Katie He on Towards Data Science, https://towardsdatascience.com/music-in-python-2f054deb41f4

This notebook is divided into three parts:

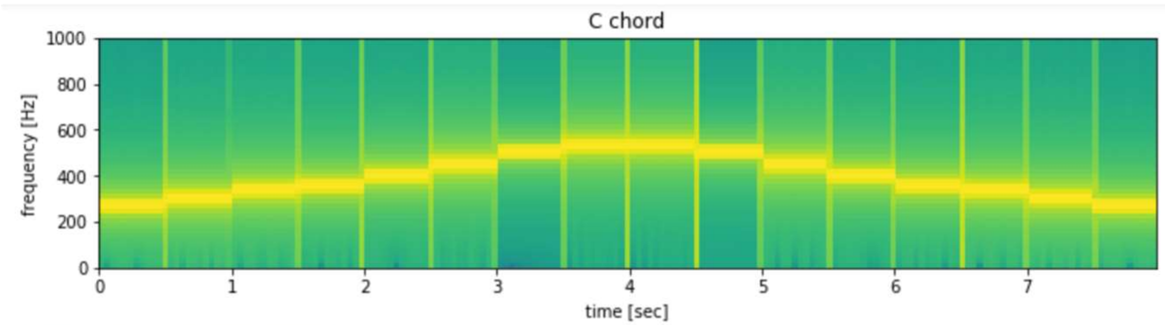
- use signal processing to analyze prominent signals in the song Bohemian Rhapsody by Queen.
- filter out higher frequencies of the song, analyze, and listen to it again.
- create audio of C chord (C major scale) using 8 single-frequency sine waves. Compare the spectrograms of C chord and the song.

You might need to install pygame first (under the Anaconda Prompt):

- pip install pygame

In [1]:
```
import numpy as np
import time
from matplotlib import pyplot as plt
import pygame
```

time and frequency representation: spectrogram



Wolfgang Amadeus Mozart
(Salzburg, 27 January 1756 – Wenen, 5 December 1791)

frequencies [Hz]: 262, 294, 330, 349, 392, 440, 494, 523

actually a time-frequency diagram (spectrogram)

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Phyphox-app demo (smartphone)

you do own a very nice collection of sensors ….

phyphox
physical phone experiments

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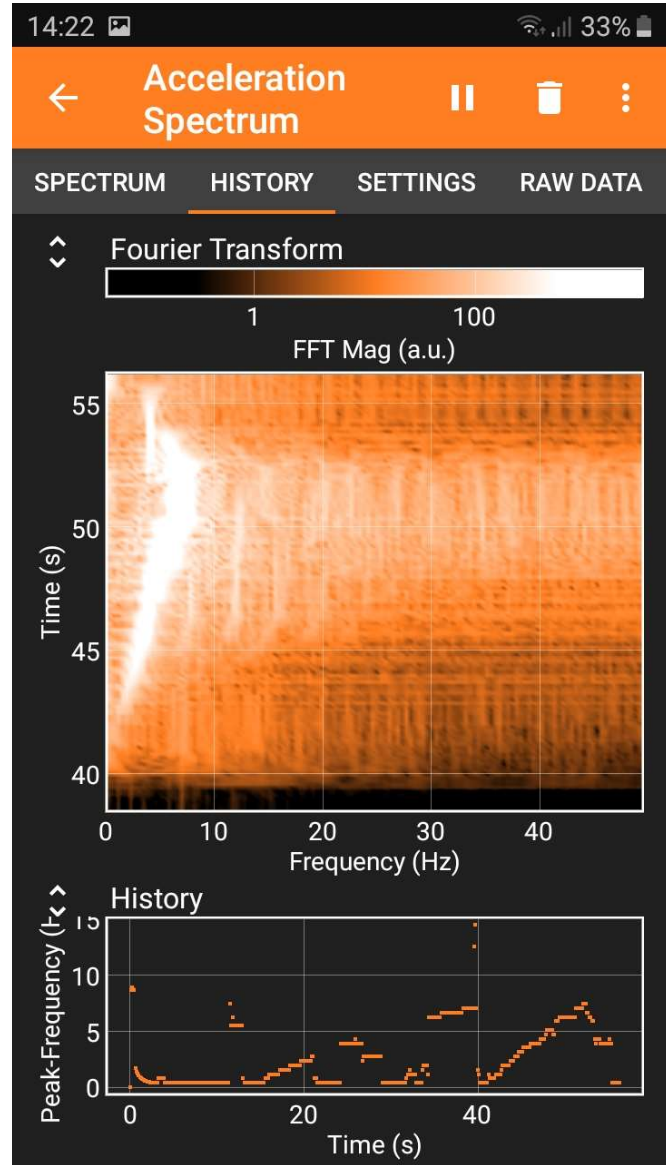

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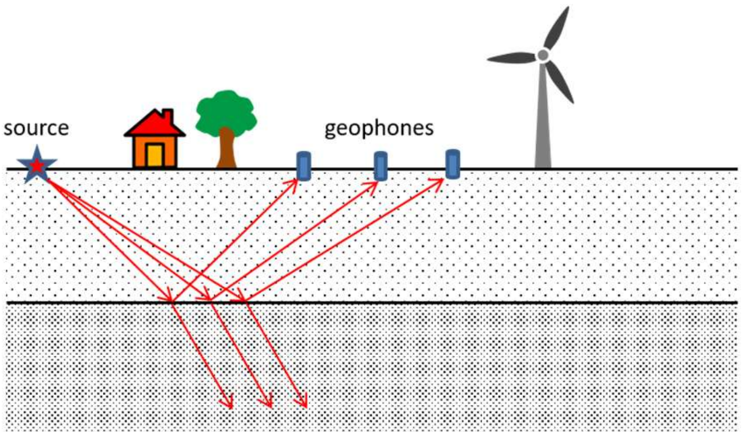
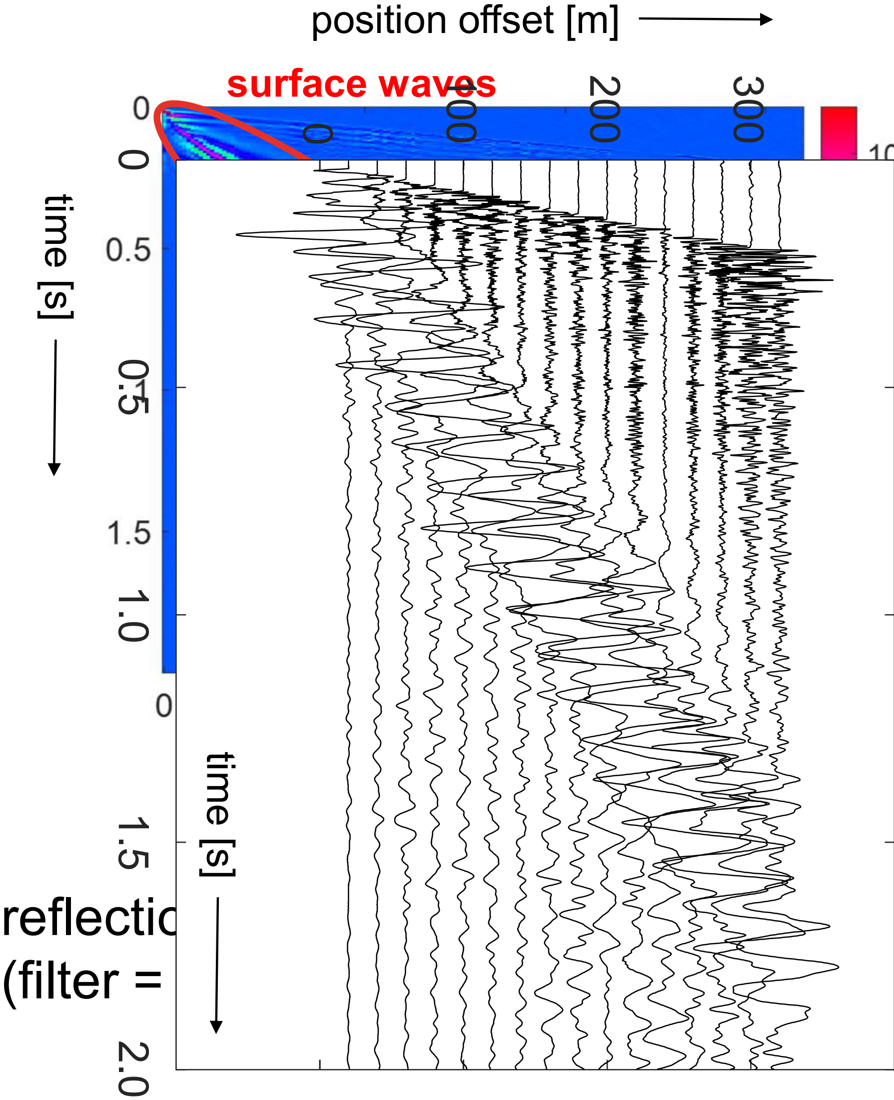
seismic reflection





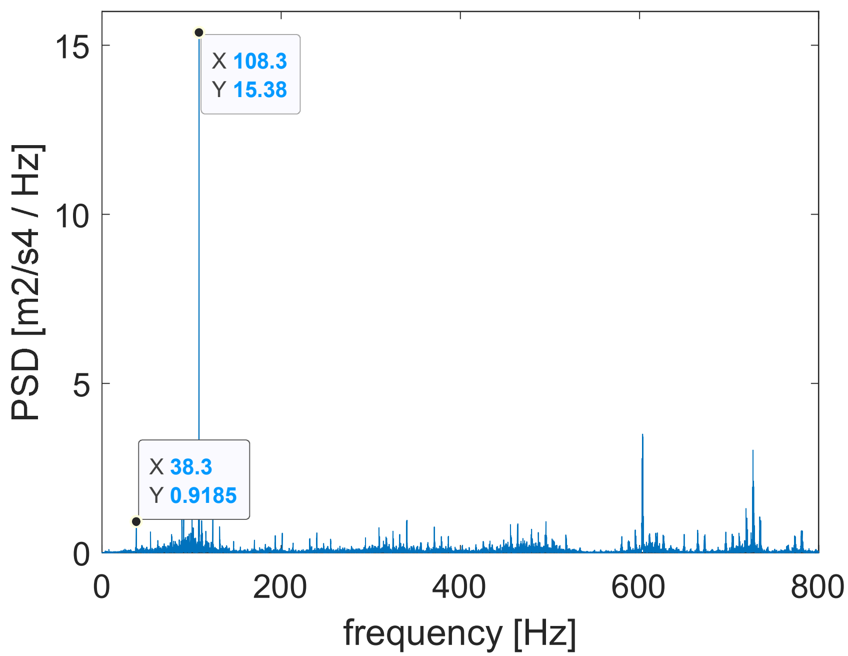
reflecti ... ecords
(filter = ...)

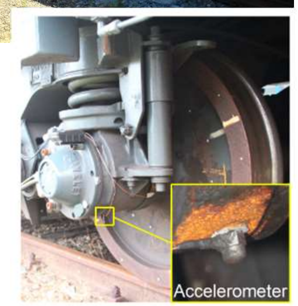
example by Guy Drijkoningen

spectral analysis in railway-engineering

using DFT, compute and visualize magnitude (amplitude) or power spectrum

analyzing signals: what **frequencies** do impact my structure,
and with what **amplitude/power**?

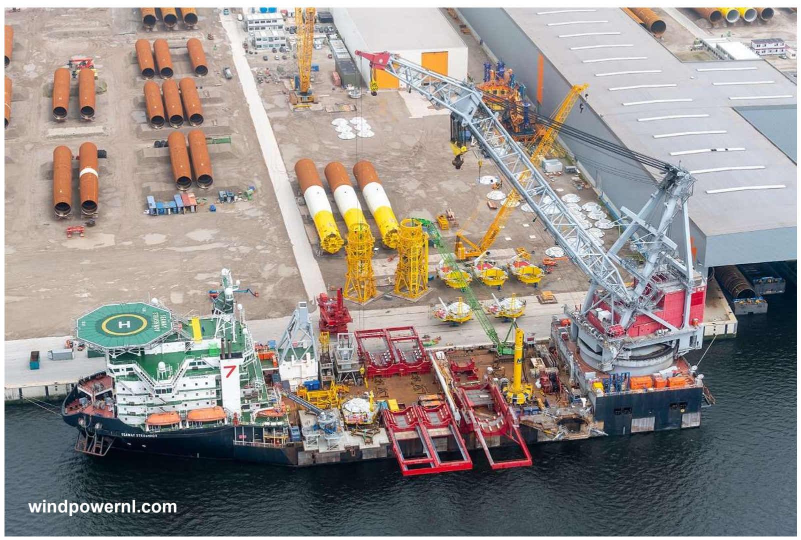




example by Chen Shen

driving down mono-piles

installing offshore wind-turbines:
hammering it down …



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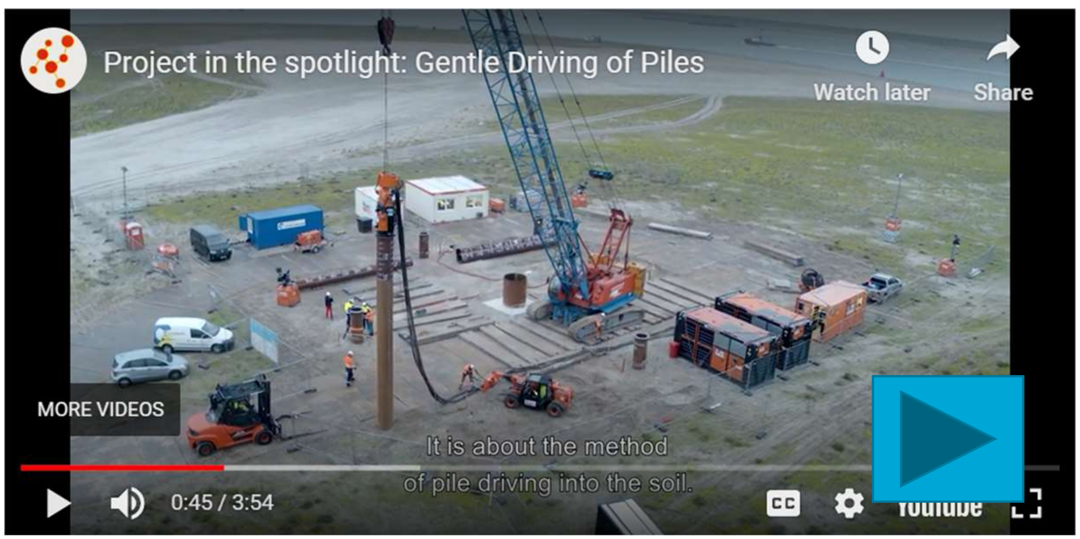
Gentle Driving of Piles (GDP)

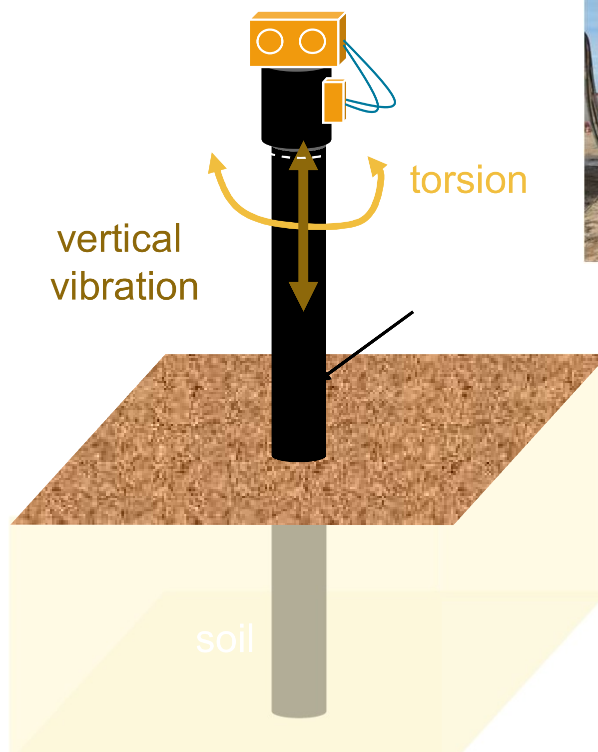
do it differently:
simultaneously apply low- and high-frequency vibrators, exciting two different modes of motion of the monopiles

https://grow-offshorewind.nl/project/gentle-driving-of-piles



GDP shaker

- combination of vibro hammer with torsional shaker
- torsional shaking as main driving mechanism
  - → avoids expansion due to driving
  - → less energy required to drive pile
- significant noise reduction compared to impact driving



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GDP project: experiment at Maasvlakte

comparing: impact hammer IP, vertical (vibro) hammering and GDP (torsional+vibro)

strain FBG technology
accelerometer MEMS

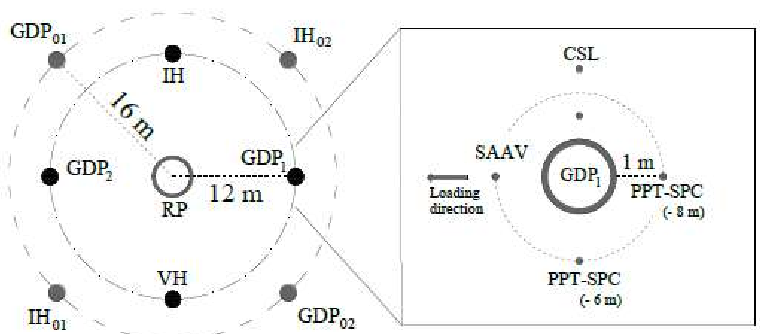
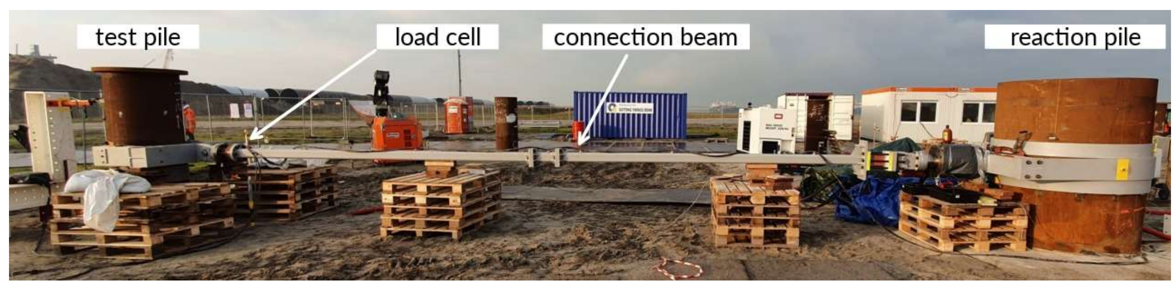
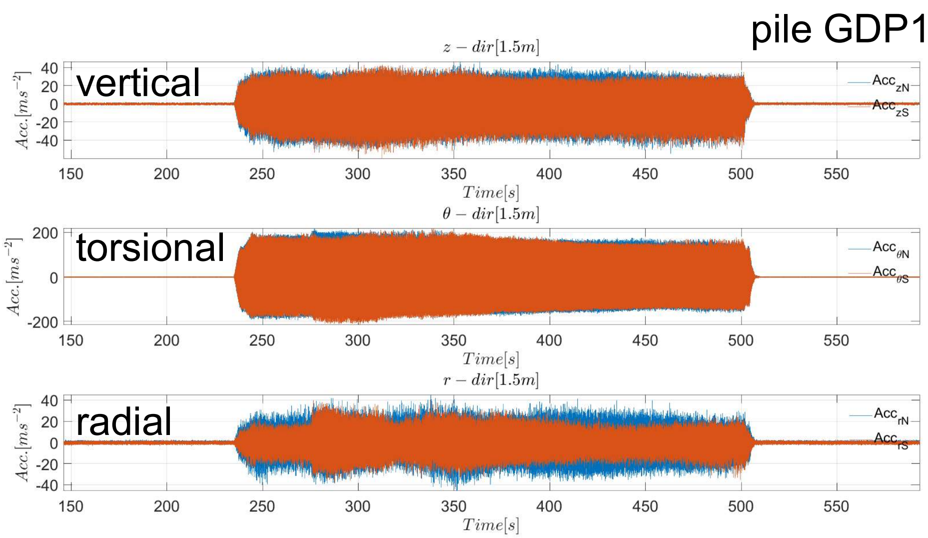


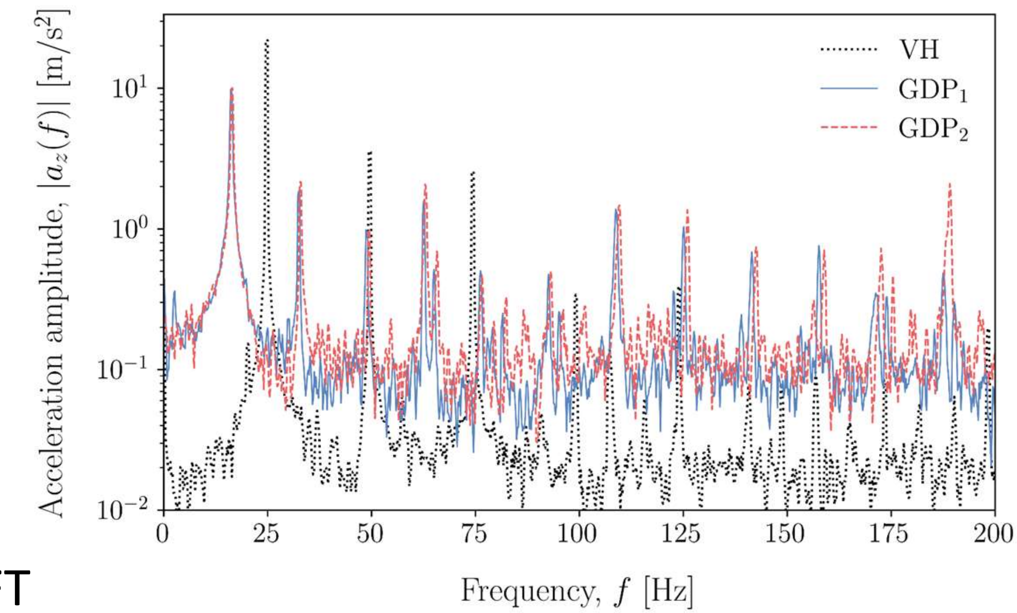
Figure 1.1: Instrumentation of a GDP pile.



GDP signal analysis

example by Sergio Sánchez Gómez





FFT

acceleration in time domain

acceleration in frequency domain for vertical direction

Sergio S. Gómez, Athanasios Tsetas, Andrei V. Metrikine,
Energy flux analysis for quantification of vibratory pile driving efficiency,
Journal of Sound and Vibration, Volume 541, 2022,
https://doi.org/10.1016/j.jsv.2022.117299.

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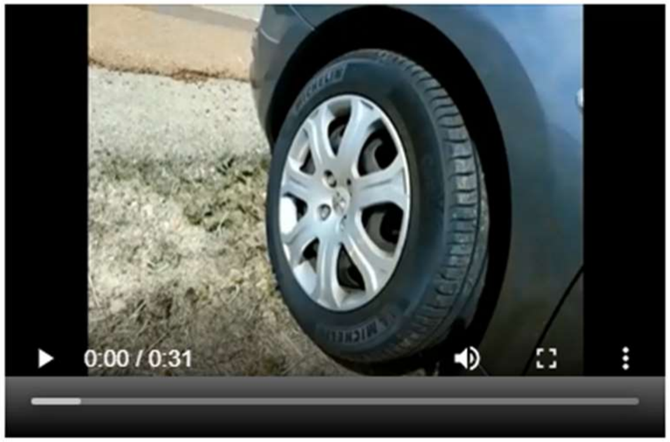
sampling – aliasing / wheel rotation movie

theory for continuous-time signals, in practice work with discrete time signals

- 30 frames per second (fps)
- periodic signal: 7 identical spokes in this wheel



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sampling – aliasing / imaging

2D signal (spatial domain, instead of temporal domain), sampling repetitive structure

sub-sampled image
→ Moiré pattern

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original digital image, 13 Mpixel

MUDE week 2.3 material

MUDE textbook – theory, derivations, in a natural order (6 chapters, each supplemented by a video ~ 10 min)

3 worked examples: pen+paper-exercise (SP-problem solving – chapters 1-3)
1 simple **Jupyter Notebook**: to demonstrate Fourier series (experience)

1 quiz on sampling (chapter 4)

workshop (Wed): Jupyter Notebook (DFT)

group assignment (Fri): analysing signals in frequency domain, in Python (synthetic, cantilever beam, sea-level) – hand-in .ipynb Notebook (for **grading**); 10 tasks (last one optional) – no separate md report

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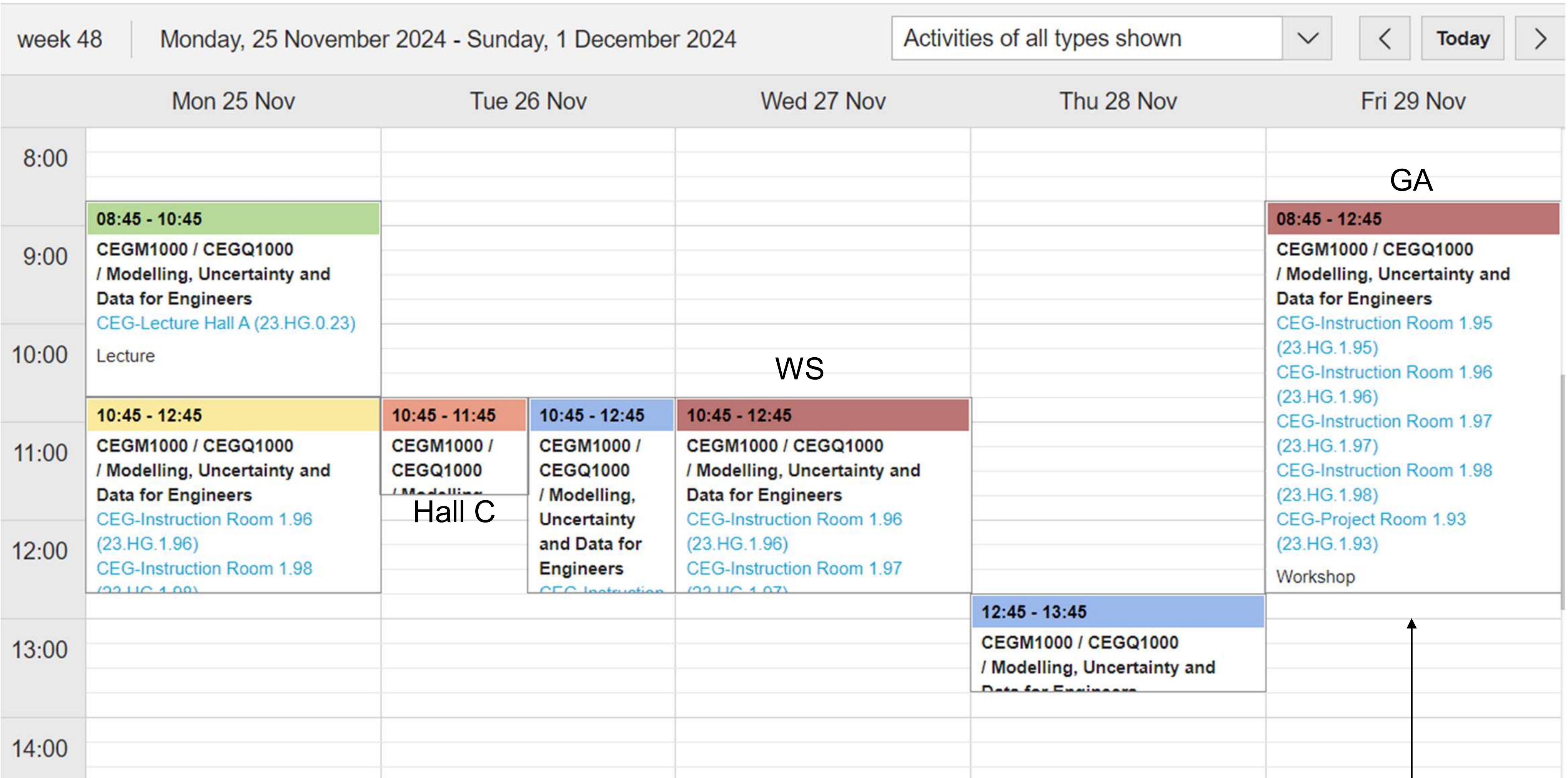

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Note: do not distribute the tasks

MUDE week 2.3 journey

learning objective:
understanding of, and insight in analysing signals, in particular in frequency domain

proofs and derivations will <u>not</u> be asked for in exam; instead, you need to be able to **apply** the theory to actual problems (problem solving), and **interpret** the results (as obtained with a Python Notebook)

no need to memorize equations

exam: focus on chapter 4 (sampling) and chapter 5 (DFT), Notebook on DFT (Wed), and in particular the questions in the group assignment (Fri)

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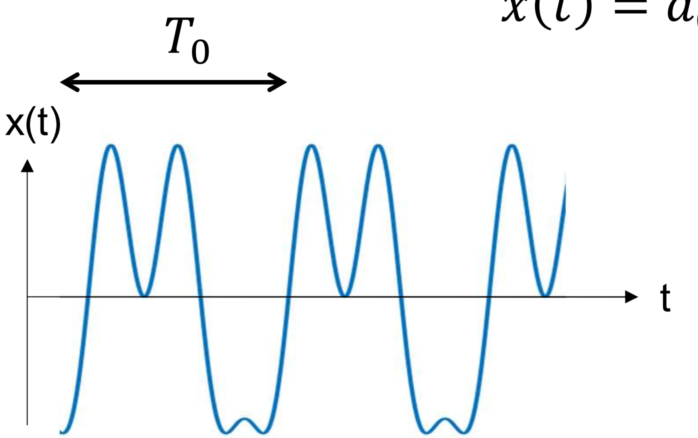
Fourier Series

express **periodic** signal $x(t)$, with period $T_0 = \frac{1}{f_0}$, as sum of harmonically related cosines and sines:

$$x(t) = a_0 + \sum_{k=1}^{k=\infty} a_k \cos(2\pi k f_0 t) + \sum_{k=1}^{k=\infty} b_k \sin(2\pi k f_0 t) \qquad k \in \mathbb{N}^+$$

real Fourier Series



continuous time $t \in \mathbb{R}$

$$e^{j2\pi f_0 t} = \cos(2\pi f_0 t) + j\sin(2\pi f_0 t)$$

$$x(t) = \sum_{k=-\infty}^{k=\infty} X_k e^{j2\pi k f_0 t} \qquad k \in \mathbb{Z}$$

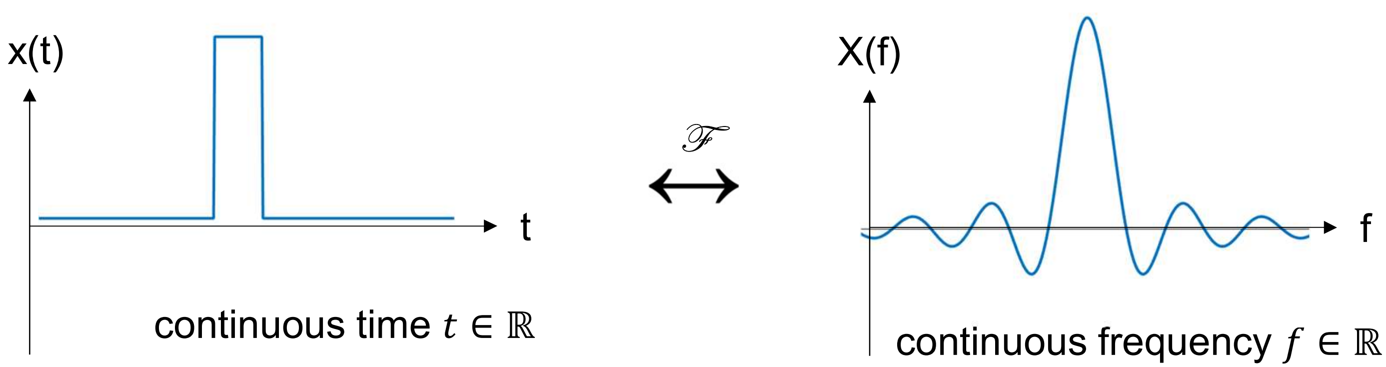
complex exponential Fourier series (double sided)

Fourier transform

express **a-periodic** signal $x(t)$, as integral over frequency f:

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t} df \qquad f \in \mathbb{R}$$

$$e^{j2\pi f t} = \cos(2\pi f t) + j \sin(2\pi f t)$$



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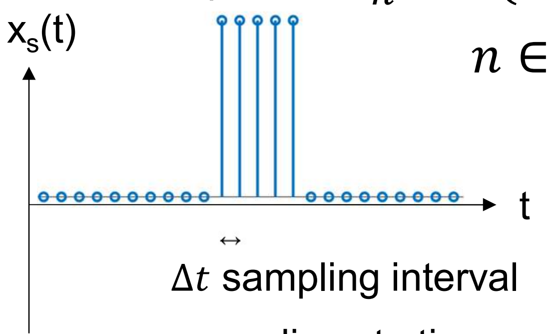
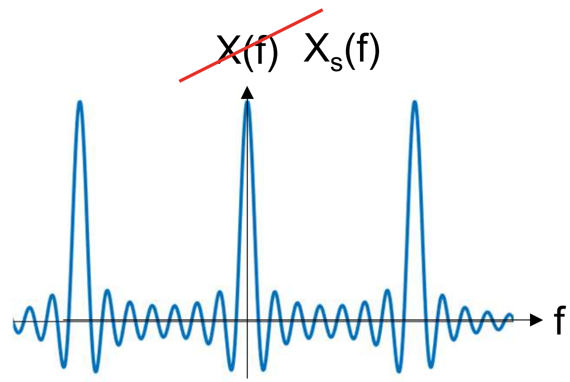
sampling → discrete time

sequence $x_n = x(n\Delta t)$

$n \in \mathbb{Z}$



$\overset{\mathscr{F}}{\longleftrightarrow}$



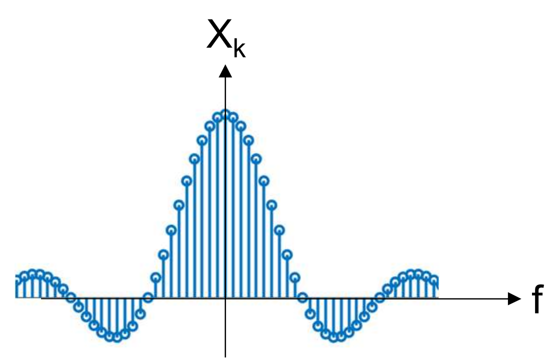
continuous frequency $f \in \mathbb{R}$

sampling in time domain generates **copies** of $X(f)$ in frequency domain

<u>Discrete Time</u> Fourier Transform (DTFT)

sample frequency domain: $k\Delta f$

$k \in \mathbb{Z}$



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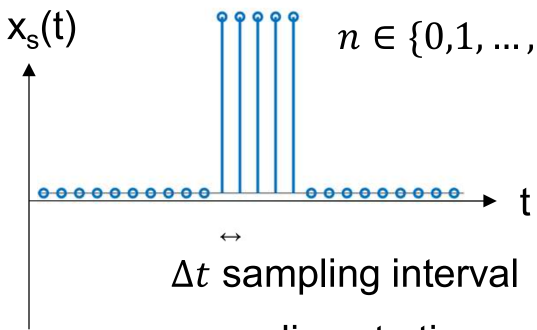
Discrete Fourier Transform (DFT)

sequence $x_n = x(n\Delta t)$

$n \in \{0,1,\ldots,N-1\}$



Δt sampling interval

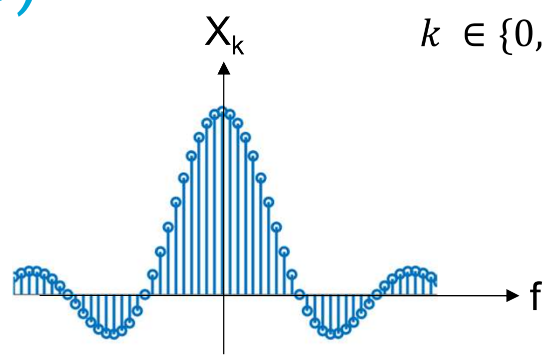
discrete time t

fft

↔

sequence $X_k = X_s(k\Delta f)$

$k \in \{0,1,\ldots,N-1\}$



0.3606	-0.1107 - 0.0630i
0.3679	-0.1081 - 0.0623i
0.3753	-0.1055 - 0.0615i
0.3827	-0.1030 - 0.0608i
0.3903	-0.1006 - 0.0601i
0.3979	-0.0983 - 0.0594i
0.4056	-0.0960 - 0.0587i
0.4133	-0.0938 - 0.0580i
0.4211	-0.0916 - 0.0573i
0.4290	-0.0895 - 0.0567i

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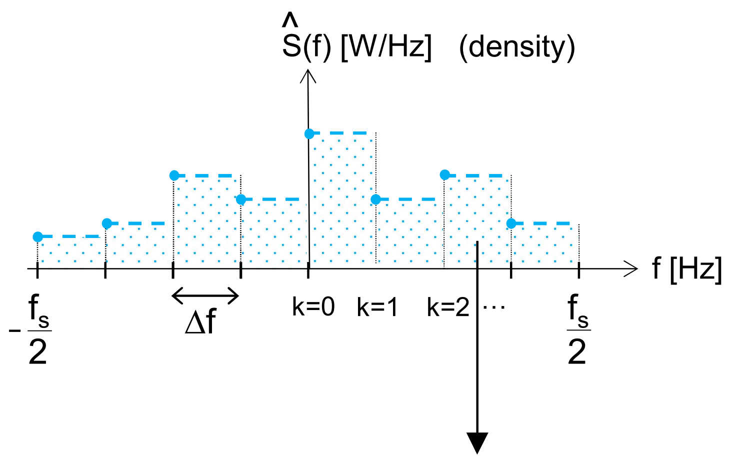
periodogram

estimate for Power Spectral Density (PSD) of signal $x(t)$: $\hat{S}(k\Delta f) = \frac{1}{T}|X_k|^2$



shows how power of signal is distributed over different frequencies

signal power: $P = \int_{-\infty}^{\infty} S(f)df$

product $\Delta f\ S(k\Delta f)$ is contribution by frequency <u>band</u> with width Δf, at frequency $f = k\Delta f$, to power P of signal

Fourier transform - history

Jean-Baptiste Joseph Fourier 1768 - 1830

Theoria Interpolationis – CF Gauss

Sit X functio arcus indeterminati x huius formae

$$\alpha + \alpha' \cos x + \alpha'' \cos 2x + \alpha''' \cos 3x + \text{etc.}$$
$$+ \beta' \sin x + \beta'' \sin 2x + \beta''' \sin 3x + \text{etc.}$$

quae non excurrat in infinitum, sed cum $\cos mx$ et $\sin mx$ abrumpatur, ita ut multitudo coëfficientium (incognitorum) sit $2m+1$. Pro totidem valoribus diversis ipsius x, puta a, b, c, d etc. dati sint valores respondentes functionis X puta $A, B, C, D \ldots$ (Ceterum valores ipsius x, quorum differentia est peripheria integra sive eius multiplum, manifesto hic pro diversis haberi nequeunt). Ex

Carl Friedrich Gauss 1777-1855

Leonhard Euler 1707-1783
Alexis-Claude Clairaut 1713 -1765
Daniel Bernoulli (1700-1782)
Joseph Louis Lagrange (1736-1813)

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Modelling, Uncertainty and Data for Engineers